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Teaching and learning algebra word problems

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Anne Lawrence

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ABSTRACT

This study reports on a classroom design experiment into the teaching and learning of algebra word problems. The study was set in the mathematics department of a co-educational secondary school, and involved two teachers and 30 Year 12 students. The teachers and the researcher worked collaboratively to design and implement an intervention that focused explicitly on translation between word problems and algebra.

Two issues were considered: the impact of the intervention on students, and the impact of the study on teachers. Students' responses to classroom activities, supported by individual student interviews, were used to examine their approaches to solving algebra word problems. Video-stimulated focus group interviews explored students' responses to classroom activities, and informed the ongoing planning and implementation of classroom activities. Data about the impact on teachers' understandings, beliefs and practices was gathered through individual interviews and classroom observations as well as the ongoing dialogue of the research team.

The most significant impact on students related to their understandings of algebra as a tool. Some students were able to combine their new-found translation skills with algebraic manipulation skills to solve word problems algebraically. However, other students had difficulties at various stages of the translation process. Factors identified as supporting student learning included explicit objectives and clarity around what was to be learnt, the opportunity for students to engage in conversations about their thinking and to practise translating between verbal and symbolic forms, structured progression of learning tasks, time to consolidate understandings, and, a heuristic for problem solving.

Participation in the project impacted on teachers in two ways: firstly, with regards to the immediate intervention of teaching algebra; and secondly, with regards to teaching strategies for mathematics in general. Translation activities provided a tool for teachers to engage students in mathematical discussion, enabling them to elicit and build on student thinking. As teachers developed new understandings about how their students approached word problems they gained insight into the importance of selecting problems for which students needed to use algebra. However, teachers experienced difficulty designing quality instructional activities, including algebra word problems, that pressed for algebraic thinking. The focus on translation within the study encouraged a shift in teacher practice away from a skills-focus toward a problem-focus.

Whilst it was apparent that instructional focus on translation shifted teachers and students away from an emphasis on procedure, it was equally clear that translation alone is insufficient as an intervention. Students need both procedural and relational understandings to develop an understanding of the use of algebra as a tool to solve word problems. Students also need to develop fluency with a range of strategies, including algebra, in order to be able to select appropriate strategies to solve particular problems. This study affirmed for teachers that teaching with a focus on understanding can provide an effective and efficient method for increasing students' motivation, interest and success.

PREFACE AND ACKNOWLEDGEMENTS

This thesis was precipitated by teachers' expressed need to improve students' use of algebra to solve word problems. The study began to form in my mind when I became involved with school-wide professional development associated with literacy. I became aware that teachers in mathematics departments had particular needs associated with literacy. Teachers were concerned about the literacy demands of word problems within mathematics. Initial exploration around this topic led me to realise that this was a widespread concern for secondary teachers, which was in part fuelled by changes associated with the introduction of NCEA. Given the emphasis on word problems within high stakes assessment, it surprised me that there was little research information available on the impact of word problems on secondary students. The introduction of NCEA had also served to highlight national concerns about algebra, and so I decided to take the opportunity, in the final stages of my degree, to explore a topic that appeared important and timely.

I would like to acknowledge and thank the many people who made this study possible. Associate Professor Glenda Anthony, my main supervisor, who provided continuing interest and invaluable professional support. My thanks are extended to Brenda Bicknell, my second supervisor, who gave positive and encouraging feedback throughout the study.

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CHAPTER 1

INTRODUCTION

Mathematical literacy is the ability to formulate and solve mathematical problems in real life situations. This type of literacy is a foundation for participation as a reflective citizen in democracy and in occupational life. (Comparative Education Research Unit, December 2004)

1.1 Background

This study seeks to improve classroom practice by informing teachers' beliefs and knowledge about the teaching and learning of algebra word problems. In seeking answers to the question about classroom experiences that will enable students to solve algebra word problems more effectively, the teachers in this study collaborated with myself as researcher to engage in exploration of their own classroom practice. They were keen to participate in this project because they wanted to improve their classroom practices and student outcomes.

Motivation for this project arose directly out of my advisory work with mathematics teachers in New Zealand secondary schools. Specifically, questions about changing teacher practice arose from my engagement with teachers in professional development programmes. Questions about students' solving of algebra word problems came from classroom practitioners who identified an increasing emphasis on word problems in external assessments. Teachers were motivated by the requirement in national assessments for students to solve word problems by writing and solving algebraic equations. Concerned about students' difficulties, teachers wanted to know how they could improve the way they taught students to use algebra as a tool to solve algebra word problems. Given the significance of word problems in high stakes assessment in New Zealand there is a need for research on specific methods for teaching students to solve word problems. Although word problems are important within other domains of mathematics, it is particularly in the algebra strand that word problems form a barrier to student progress in secondary school.

The importance of algebra is stressed by Moses and Cobb (2001) who argue that algebra is the “key to the future of disenfranchised communities” (p. 5) because it is not only the gatekeeper to higher mathematics as well as “the gatekeeper for citizenship; and people who don’t have it are like the people who couldn’t read and write in the industrial age” (p. 14). However, despite the importance of algebra, it has proven to be a serious stumbling block for many students. Difficulties experienced by students at secondary school have contributed to the recent research and curricula emphasis on algebraic thinking and reasoning in the elementary years, with experiences in Pre-algebra and Early Algebra seen as critical for building the understandings and skills of formal algebra (Kieran, 2006; Stephens, 2006). This emphasis, however, does not abrogate responsibility for improving the teaching of formal algebra in the later years, which is the focus of this study.

Internationally, there has been a significant change in emphasis in school mathematics over the last three decades. Mathematics reform documents support an inquiry approach to teaching; students working in inquiry-based classrooms engage in mathematical discourse, sharing and refining their mathematical understandings by participating in learning communities. Inquiry classrooms involve a shift away from students’ acquisition of procedural proficiencies to the development of their abilities to solve problems in meaningful contexts. Aligned with the focus on mathematical discourse and contextual problems is a growing awareness of the importance of language factors in the teaching and learning of mathematics (Curcio, 2004; Dowling, 2001; Ellerton & Clarkson, 1996; MacGregor & Price, 2002; Meaney, 2006).

Within New Zealand, word or story problems are emphasised in the mathematics curriculum (Ministry of Education, 1992) and feature prominently in high-stakes assessment. Although there is debate about the merits of assessment as a driving force for teaching, it is clear that what is measured in high-stakes assessments does influence what is taught in classrooms (Clarke, 2005). In New Zealand, the national assessment system has undergone significant changes since 2003 when the norm-referenced system was replaced by a standards-based system. Students now work towards a National Certificate of Educational Achievement (NCEA) which is assessed by performance against criteria defined by Achievement Standards. There are three levels of Achievement Standards,

and four categories of performance: not achieved, achieved, achieved with merit or achieved with excellence.

There are two achievement standards that focus on the use of algebra, one at Level One and one at Level Two (see Appendix A). Although schools can set their own course entry requirements, Level One Algebra is a common pre-requisite for Year 12 mathematics and Level Two Algebra is a common pre-requisite for Year 13 calculus. Both the algebra standards include the solving of algebraic word problems. Explanatory notes from the New Zealand Qualifications Authority (NZQA, 2005d) detailing the requirements for achieving the standards specify that students who achieve the standard are able to “use algebraic strategies to investigate and solve problems...Problems will involve modelling by forming and solving appropriate equations, and interpretation in context” (NZQA, 2005b, p. 2). Contextual problems are emphasised by the exam specifications which state: “Questions providing candidates with opportunities for achievement with merit and achievement with excellence will be set in real-life contexts” (NZQA, 2005d, p. 1). For the questions involving the solving of word problems, the assessment schedule states that a student “must form equations...at least one equation” (NZQA, 2005c, p. 2). According to this schedule non-algebraic methods are not recognised as valid solution methods.

National assessment results reflect poor achievement rates on the algebra standards. The most recent examiner’s report highlighted students’ difficulties with these achievement standards (NZQA, 2006b, 2006c). As an adviser, I facilitated NCEA professional development and facilitated workshops for teachers. Teachers suggested that student difficulties were exacerbated by three aspects of the algebra achievement standards: the emphasis on contextual problems; the writing of algebraic equations; and the need to solve these equations algebraically. They sought support to address these aspects.

The literature (e.g., Bennett, 2002; Koedinger & Nathan, 2004) suggests that writing and solving of equations is likely to be a significant cause of difficulty. International research indicates that secondary students tend to use informal methods even when they have been taught more formal algebraic methods. This is a concern beyond that of the achievement standards assessments, as reliance on informal methods hinders progress in higher mathematics. “There are important ideas that can best be communicated by using the symbols of algebra” (Foreman, 1997, p. 161).

The process of solving algebraic word problems can be viewed in terms of a comprehension stage and a solution stage. Writing equations involves translating from words into algebra as a part of the comprehension stage. This translation stage was a key focus for the project. The literature proposes a range of teaching practices to address the process of translation between algebraic and verbal representations, but research is needed to trial methods within the New Zealand secondary school context.

Alongside the focus on teaching practice, teacher learning was an important focus of this research. Recent studies have highlighted the importance of the teacher's role for student learning. Alton-Lee's (2003) *Quality teaching for diverse students in schooling: Best evidence synthesis* argued that "quality teaching is optimised when teachers have a good understanding of and are responsive to, the student learning processes involved" (p. 45). "Teachers' beliefs and knowledge... have a profound effect on the decisions they make regarding instruction" (Fennema, Sowder, & Carpenter, 1999, p. 10). However, Timperley, Fung, Wilson, and Barrar (2006) argue that understanding the learning processes involved in changing teacher practice is a neglected area of research.

1.2 Research Objectives

This study has a dual focus—teacher learning and student learning. It aims to create new insights and knowledge about effective teacher practice in relation to the teaching of algebra. It trials specific teaching strategies with the aim of enhancing students' use of algebra as a tool that replaces informal strategies in solving word problems. This research also aims to address the building of teachers' pedagogical content knowledge through the exploration of effective teaching strategies. The intervention involved the development and implementation of instructional activities that explicitly focus on translating between verbal and symbolic representations of algebra word problems.

Data is generated to address the following research questions:

1. In what ways does the introduction of explicit teaching activities that support the translation processes used to solve algebra word problems impact on student learning processes and outcomes?
2. How does participation in the classroom experiment impact on teachers' pedagogical practices, knowledge and beliefs?

1.3 Thesis Overview

Chapter 2 provides a review of the literature in the field and provides a background from which this project can be viewed. It summarises relevant and essential findings on the issues of mathematical pedagogical content knowledge, formal school algebra and student difficulties with algebra word problems and translation, the various pathways followed by students in solving algebra word problems, and the implications of these issues for instruction.

Chapter 3 presents a discussion of the methodology for the study with reference to effective approaches for teacher change. This chapter also includes the data generation methods and an outline of the project schedule.

In Chapter 4, the teaching activities used in the project are described. The results are reported and discussed in Chapters 5 and 6. Chapter 5 reports on the processes students used to solve algebra word problems. Chapter 6 presents and discusses teachers' views and responses to their involvement in the project.

The final chapter addresses the research questions, summarises key themes emerging from the project, discusses limitations of the project, and makes suggestions for further research.